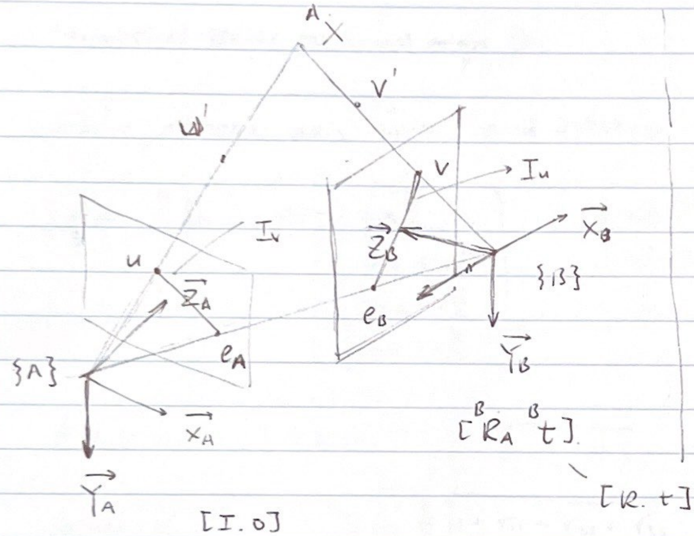
EPIPOLAR GEOMETRY



- Camera Model

- $\lambda \hat{u} = K_A [I\ 0] \begin{pmatrix} {}^A X \\ 1 \end{pmatrix}$

  $\lambda v = K_B [R\ t] \begin{pmatrix} {}^A X \\ 1 \end{pmatrix}$

- Define 3D point $\hat{U}$ $\hat{V}$.

  $\hat{U} = \lambda (K_A [I\ 0])^{-1} u$

  $\hat{V} = \lambda (K_B [R\ t])^{-1} v$

The descriibtion of epipolar geometry:

A. B. $\hat{U}$ $\hat{V}$ is in the same plane.

$\boxed{\cancel{\vec{BV} \cdot (\vec{AU} \times \vec{AB}) = 0}}$

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point A: 0.

point B: $-R^{-1} t$

point $\hat{U}$: $\lambda_u K_A^{-1} u$.

point $\hat{V}$: $-R^{-1} t + \lambda_v [R^{-1}\ \ -R^{-1} t] K_B^{-1} v$

$$\lambda_v [R^{-1}\ -R^{-1} t] K_B^{-1} v \cdot \left[(\lambda_u K_A^{-1} u) \times (-R^{-1} t) \right] = 0.$$

$\vec{a} \cdot (\vec{a} \times \vec{b})$

$$(R^{-1} K_B^{-1} v - R^{-1} t) \cdot (K_A^{-1} [u]_\times (-R^{-1} t)) = 0$$

$$R^{-1} K_B^{-1} v \cdot K_A^{-1} [u]_\times R^{-1} t = 0.$$

$$\left(\cancel{\lambda_u} K_A^{-1} u\right)^T \cdot \left[\left(-R^{-1} t\right) \times \cancel{\lambda_v}\left(R^{-1} K_B^{-1} v - \cancel{R^{-1} t}\right)\right] = 0.$$

$$= \left(K_A^{-1} u\right)^T \left[\left(R^{-1} t\right) \times \left(R^{-1} K_B^{-1} v\right)\right] = 0$$

$(Ma) \times (Mb) = \det(M)(M^{-1})^T (a \times b)$

$$\left(K_A^{-1} u\right)^T \; \det(R) \cdot \left((R^{-1})^{-1}\right)^T \cdot \left[t \times K_B^{-1} v \right] = 0$$

$$\left(K_A^{-1} u\right)^T R^{-1} [t]_\times \cdot K_B^{-1} v = 0.$$

$$[t]_\times = \begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix}$$

$$\boxed{v^T \underbrace{K_B^{-T} [t]_\times R \cdot K_A^{-1}}_{F} u = 0.}$$

F: Fundamental Matrix. $\boxed{v^T F \cdot u = 0.}$ 8 correspondences

$I_u = F u$. $I_v = F^T \cdot v$. $u^T I_v = 0$. $v^T \cdot I_u = 0$.

▷ Essential Matrix: $\boxed{F = K_B^{-T} \underbrace{[t]_\times R}_{E} K_A^{-1}}$ F: rank(2)

$$\boxed{E = K_B^T F K_A.}$$ $E = [t]_\times R$

Property: $E = [t]_\times R = U D V^T = U \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} V^T$

▷ Camera Pose Estimation: $t = \pm \text{null}(E^T) = \pm \text{null}(R^{-1}[t]_\times^T)$