TRIANGULATION

Suppose $[R.t]$ is given. now calculate ${}^{A}X$

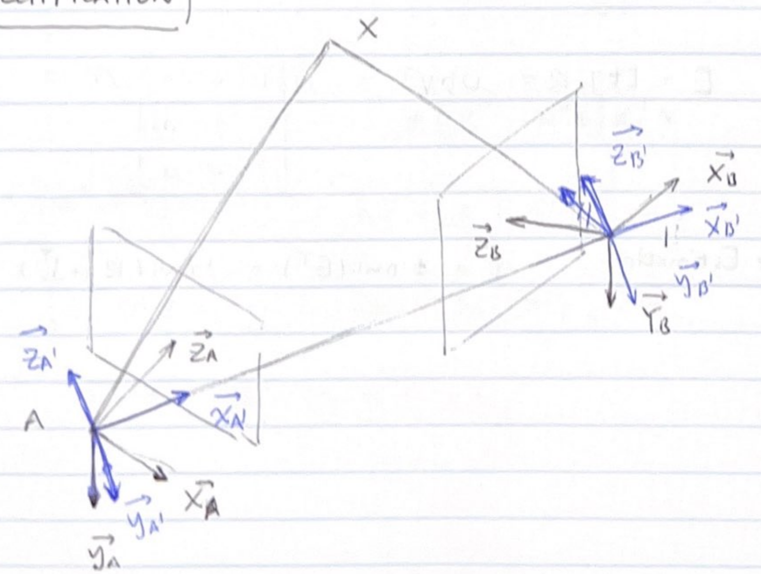
$$\lambda_1 \begin{bmatrix} u \\ 1 \end{bmatrix} = K_A [I.\ 0]\ {}^{A}X \longrightarrow \begin{bmatrix} u \\ 1 \end{bmatrix} \times \underbrace{(K_A [I\ \ 0])}_{P_A}\ {}^{A}X = 0$$

$$\lambda_2 \begin{bmatrix} v \\ 1 \end{bmatrix} = K_B [R.t]\ {}^{A}X \longrightarrow \begin{bmatrix} v \\ 1 \end{bmatrix} \times \underbrace{(K_B [R\ \ t])}_{P_B}\ {}^{A}X = 0$$

$$\underbrace{\begin{bmatrix} [u]_\times P_A \\ [v]_\times P_B \end{bmatrix}}_{4\times 4} \begin{matrix} {}^{A}X = 0 \\ {}^{A}X = 0. \end{matrix} \longrightarrow \text{rank}(2) \longrightarrow A X = 0.$$

▷ Camera Pose Disambiguation. (Cheirality) : $\underline{{}^{A}X_z} > 0$ & ${}^{B}X_z > 0$

RECTIFICATION



Rotate camera in place from A to A'. $\boxed{R_{rect} = {}^{A'}R_A}$

$$\lambda_1 \begin{pmatrix} u \\ 1 \end{pmatrix} = K_A [I \;\; 0]\, {}^{A}X$$

$${}^{A}R_{A'} = [\, r_x \;\; r_y \;\; r_z \,]$$

$$r_x = {}^{A}t / \| {}^{A}t \|$$

$$r_z = \vec{z}_A - (\vec{z}_A \cdot \vec{x}_{A'}) \cdot \vec{x}_{A'}$$

$$r_y = r_x \times r_z.$$

$$\lambda_2 \begin{pmatrix} u' \\ 1 \end{pmatrix} = K_A \, \underbrace{{}^{A'}R_A}_{R_{rect}} \, {}^{A}X$$

$$\begin{pmatrix} u' \\ 1 \end{pmatrix} = \underbrace{K_A R_{rect} K_A^{-1}}_{H_A:\ \text{homography}} \begin{pmatrix} u \\ 1 \end{pmatrix}$$

$$\boxed{H_A = K_A R_{rect} K_A^{-1}}$$

Rotate camera in place from B to B'.

$$\lambda_1 \begin{pmatrix} u \\ 1 \end{pmatrix} = K_A \, {}^{B}R_A \, ({}^{A}X - {}^{A}C)$$

camera position ${}^{A}C = -{}^{B}R_A^{-1}\, {}^{B}t$

$$\lambda_2 \begin{pmatrix} u' \\ 1 \end{pmatrix} = K_B \, {}^{B'}R_B \, {}^{B}R_A \, ({}^{A}X - {}^{A}C) \rightarrow \boxed{H_B = K_B R_{rect} R \, K_B^{-1}}$$

What if the camera does not rotate in place. For A frame for example,

$$\lambda_2 \begin{pmatrix} u' \\ 1 \end{pmatrix} = K_A \left[{}^{A'}R_A \;\; {}^{A'}t \right] {}^{A}X = K_A \, {}^{A'}R_A \, ({}^{A}X - {}^{A}C')$$

$$= K_A \, {}^{A'}R_A \left[\lambda_1 K_A^{-1} \begin{pmatrix} u \\ 1 \end{pmatrix} - {}^{A}C' \right]$$

Homography is a function of $\lambda_2 . \lambda_1$ whereas without, ${}^{A}C' = 0$.

$$\begin{pmatrix} u' \\ 1 \end{pmatrix} = \frac{\lambda_1}{\lambda_2} \cdot \underbrace{K_A \, {}^{A'}R_A \cdot K_A^{-1}}_{H} \begin{pmatrix} u \\ 1 \end{pmatrix}$$